

Solving and Computing DSGE models in the computer (Part 2 - Using Dynare)

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Once you've learned to do the heavy lifting (i.e.: manually programming value function iterations, linear approximations, etc.) there is a very useful Matlab *add-on* called Dynare that can do most of those things for us. This option becomes especially handy when one would like to quickly test the implications of a model, without enduring the cost of programming its solution from scratch.

Over the years this compilation of Matlab routines has evolved and become extremely complete allowing for high order approximations to the policy function, Bayesian estimation of parameters as well as forecasting and handling Markov-switching models. Below are a few fundamental commands, basic conventions and examples to keep in mind when learning to use this tool. These notes were made with Dynare version 4.4.3. for Mac OS X. You should check for newer stable versions from their website <http://www.dynare.org>.

***DISCLAIMER:** I wrote these notes as a study aid for myself. They are work in progress and could be incomplete, inaccurate and even somewhat incorrect. Keep that in mind should you decide to use them. Comments and suggestions welcomed!

1 Dynare basic syntax and conventions

1.1 Some general tips:

The best and most complete reference to learn how to use Dynare is the official guide that comes with every installation file. Below are a few general guidelines:

1. To run Dynare, you must first install it and then create Matlab path to its folder.
2. Dynare files take a ".mod" extension. You can create these files in Matlab's Editor, but be sure to save them as ".mod" and not as ".m" files.
3. To run a Dynare file, simply type "dynare filename" into the command window while in Matlab. For e.g.: "dynare rbc.mod".
4. Timing convention: Dynare uses a particular timing convention that might require us to rewrite the model when creating a mod-file. Specifically, for a variable x :
 - (a) If x is decided in period t , then one writes x .
 - (b) If x is decided in period $t-1$ (e.g.: k_t), then one writes $x(-1)$.
 - (c) If x is decided in period $t+1$ (e.g.: consumption in Euler eq), then one writes $x(+1)$.

This is how Dynare understands which are the predetermined variables prior to period "t" (see example below).

5. A mod-file will not run unless every entry is followed by a semicolon.
6. When studying a model's dynamics, it is useful to study its behavior around the steady state. Dynare can solve a model's steady state numerically for us, yet for this to happen we need to provide it with an initial set of values (or guess) from where to begin. This is why solving for the analytical steady state can often be useful. Giving Dynare's solver a good starting point will hopefully enable it to find the solution faster. I will do this for every model in this document¹.

¹Moreover, Matlab includes some very powerful equation solvers. If finding the steady state analytically becomes too difficult, we can also use numerical methods outside of Dynare which the program can accommodate.

1.2 Example - Timing Convention

Assume we have the following infinite horizon problem:

$$\begin{aligned} \max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} U &= E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma} - 1}{1-\gamma} \\ \text{s.t.} \quad &: z_t k_t^\alpha \geq k_{t+1} - (1-\delta)k_t + c_t \\ &: \ln z_t = \rho \ln z_{t-1} + \epsilon_t \end{aligned}$$

where $\epsilon_t \sim N(0,1)$. The first order conditions to this problem can be characterized by three non-linear difference equations and a transversality condition:

$$\begin{aligned} c_t^{-\gamma} &= \beta E_t c_{t+1}^{-\gamma} \left[\alpha z_{t+1} k_{t+1}^{\alpha-1} + (1-\delta) \right] \\ z_t k_t^\alpha &= k_{t+1} - (1-\delta)k_t + c_t \\ \ln z_t &= \rho \ln z_{t-1} + \epsilon_t \\ 0 &= \lim_{t \rightarrow \infty} \beta^t c_t^{-\gamma} k_{t+1} \end{aligned}$$

Additionally we have the accounting identity:

$$z_t k_t^\alpha = i_t + c_t$$

The timing convention in Dynare requires us to slightly rewrite the above equations. Specifically, it requires that predetermined variables (such as the capital stock in this example) to be as written as dated in "t-1" format. The remaining variables (c_t, z_t, i_t) are all going to be determined within that period. Some exogenously (z_t), some endogenously (i_t), and some are the agent's choice variables (c_t).

Given this, we would need to rewrite the FOCs above as:

$$\begin{aligned}
c_t^{-\gamma} &= \beta E_t c_{t+1}^{-\gamma} \left[\alpha z_{t+1} k_t^{\alpha-1} + (1 - \delta) \right] \\
z_t k_{t-1}^\alpha &= k_t - (1 - \delta) k_{t-1} + c_t \\
\ln z_t &= \rho \ln z_{t-1} + \epsilon_t \\
0 &= \lim_{t \rightarrow \infty} \beta^t c_t^{-\gamma} k_t \\
z_t k_{t-1}^\alpha &= i_t + c_t
\end{aligned}$$

A similar change is required in models with capital markets where agents may accumulate financial assets. For example if households have access to non-state contingent one period risk-less bonds, the equilibrium conditions would include the following Euler equation:

$$c_t^{-\gamma} = \beta E_t c_{t+1}^{-\gamma} (1 + r_{t+1})$$

where r_{t+1} is the return on the bond (usually 1 unit of the consumption good) bought at time "t" and held until "t+1". The key is that while the bond pays at "t+1", its return is known before hand at time "t". In other words, the value of r_{t+1} is included in the information set at time "t". Consequently we could rewrite the above as r_{t+1} being out of the expectation operator.

$$c_t^{-\gamma} = \beta (1 + r_{t+1}) E_t c_{t+1}^{-\gamma}$$

In terms of Dynare, this means that r_{t+1} is really determined at time t and is not an expected value that gets realized at $t + 1$. Hence, we would need to rewrite the above Euler equation as:

$$c_t^{-\gamma} = \beta (1 + r_t) E_t c_{t+1}^{-\gamma}$$

1.3 Writing the actual code:

Knowing a model's first order conditions is enough for us to begin getting the model into Dynare. The usual steps are as follows:

1. Declare the variables of the model:

You declare all endogenous by writing *"var"* followed by the endogenous variables names and culminating with a semicolon. You declare all exogenous variables by writing *"varexo"* and repeating the procedure above.

2. Declare the parameters of the model:

You must first declare the parameters by typing *"parameters"* followed by their names. Immediately following this command you need to specify the actual values for these parameters obtained by calibration/estimation.

3. Declare the model:

To introduce the model one must type *"model"* followed by the first order conditions, constraints, economy's identities, etc., followed by *"end"*. Two important details:

- (1) When you type in the FOCs (using Dynare's timing convention) if the variable appears dated "t", then you simply type x. If the variable appears dated "t-1", then you type x(-1) and if it is dated as "t+1", you insert x(+1).

- (2) In order to solve a model Dynare will perform a local approximation to the policy functions around the model's steady state values. Currently, the default is to compute a second order approximation. When you type the model into Dynare you may input it in levels or in logs. The latter has the nice interpretation of having impulse responses that are expressed in percentage terms deviations from ss-values. To get Dynare to do an approximation of the variables in logs, you need to specify the variables as *"exp(x)"*. This way the variable x is interpreted as the log of the variable of interest, while *exp(x)* represents its level. Last, it is also possible to provide Dynare with the linearized version of a model. In such a case, Dynare will not perform a linear approximation as it understands the model has been linearized. The relevant command is *"model(linear)"*.

4. Declare initial values for variables:

Since Dynare will perform an approximation around a model's deterministic steady state, the program will first try to compute these values using a non-linear solver. To achieve this the user must supply an initial starting point for the solver to begin searching. Each variable, endogenous and exogenous, must be initialized.

Alternatively, one may calculate the ss-values outside of Dynare. In that case you can simply supply these values as the initial values for Dynare's solver to begin working with. The relevant command is "*initial*" followed by the initial guesses of the endogenous variables and finished off by "end". This command will also be used for deterministic set-ups, yet in a different way (see examples of deterministic models).

5. Declare the variance of exogenous variables:

We begin this declaration by the word "*shocks*", followed by the variance (not the standard deviation) of the variable, followed by "end". Note that if one assumes that the disturbances are correlated, the variance-covariance matrix must also be specified (see RBC example below).

6. Determine the model's steady state:

This step can be done by Dynare by invoking the command "*steady*" or it can be supplied by the user. However, since Dynare employs a Taylor expansion technique to approximate the policy function, a steady state will always be computed.

7. Simulate model:

In order to solve for the policy functions and simulate the model we must use the command "*stoch_simul*". There are a number of important options regarding this command:

- (a) "*hp_filter = lambda*": Produces theoretical moments after HP-filtering the data. For biz-cycles we typically want to look at HP filtered moments, so this is an interesting option. The lambda is the filter's penalty parameter (e.g.: 1600 for Q, 100-400 for y, etc.).

- (b) "irf = integer": Sets the number of periods plotted in the impulse response functions. Currently, the default value is 40. No irfs are computed if set to 0.
- (c) "noprint": This will suppress the printing of any output.
- (d) "order = integer": Tells Dynare the order of the (log) approximation. The default is second order. Typing "order=1" will have it do a linear approximation.
- (e) "drop = integer": Changes the number of observations to drop from the simulations. For example, "stoch_simul(drop=1)" drops the first simulation.
- (f) "periods = integer": Dynare's default option is to produce analytical/theoretical moments of the variables. Setting periods to a number different from zero will instead have it simulate data and compute the moments from the simulations. By default, Dynare drops the first 100 values from a simulation, so you need to give it a number greater than that. For example, "stoch_simul(periods=300)" will produce moments based on 200 simulated periods.
- (g) "simul_seed": This option lets you set the seed used in the random number generator for simulations.

For example, typing "stoch_simul(nofunctions, hp_filter = 1600, order = 3, irf = 30)" will suppress the policy function output, produce analytical HP filtered moments, will do a 3rd order approximation to the policy function and will plot impulse responses for 30 periods.

Below is a complete example of what a Dynare mod-file looks like. It corresponds to a neoclassical growth model with an exogenous stochastic process². For this, as well as for all other examples in this note, you may download the corresponding mod-files from my site. To run your mod-file you must type "dynare filename.mod" while in Matlab. For this to work, you must either be in the same directory where your Dynare files are stored or you must have set the path to where your mod-files are stored.

²I tend to write parameters in capital letters and variables in lower case. In simple models this might seem excessive, but for bigger ones it helps to quickly distinguish between the two. You can thank Junior Maih for this.

```

% Dynare basic example
% Summer 2014

% Housekeeping:
clc;
clear all;

% Endogenous variables:
var y i k z c;

% Exogenous variables:
varexo e;

% Parameters:
parameters ALPHA, RHO, BETA, DELTA, GAMMA, GAMMAE;
ALPHA = 0.33;
RHO = 0.95;
BETA = 0.99;
DELTA = 0.025;
GAMMA = 1;
GAMMAE = 0.01;

% The Model:
model;
exp(c) ^ (-GAMMA) = BETA*(exp(c(+1)) ^ (-GAMMA) * (ALPHA*exp(z(+1))*exp(k) ^ (ALPHA-1) + (1-DELTA)));
exp(y) = exp(z) * exp(k(-1)) ^ (ALPHA);
exp(k) = exp(i) + (1-DELTA)*exp(k(-1));
exp(y) = exp(c) + exp(i);
z = RHO * z(-1) + e;
end;

% Declare initial Values:
initval;
k = log(29);
y = log(3);
z = 0;
c = log(2.5);
i = log(1.5);
end;

shocks;
var e = GAMMAE ^ 2;
end;

steady;

stoch_simul(hp_filter=1600, order=2, irf=40);

```

1.4 Interpreting Dynare's output:

Dynare will produce a large amount of output in the command window once it has finished running. The most important part of it are the coefficients on the policy functions as well as the matrix of moments if you are studying business cycles. An example of such matrices is copied below:

POLICY AND TRANSITION FUNCTIONS					
	y	i	k	a	c
Constant	1.103709	-0.344308	3.344571	0	0.835782
k(-1)	0.330000	-0.517541	0.962061	0	0.590408
a(-1)	0.950000	3.043702	0.076093	0.950000	0.306708
e	1.000000	3.203897	0.080097	1.000000	0.322850

THEORETICAL MOMENTS (HP filter, lambda = 1600)

VARIABLE	MEAN	STD. DEV.	VARIANCE
y	1.1037	0.0131	0.0002
i	-0.3443	0.0418	0.0017
k	3.3446	0.0037	0.0000
a	0.0000	0.0130	0.0002
c	0.8358	0.0047	0.0000

The constants in the policy function are the steady state values. The coefficients of the (modified) state space representation are understood as always. In addition Dynare produces means (steady state values), standard deviations and variances for each endogenous variables. Given the options I've chosen, all of these moments are theoretical (i.e.: not based on simulations) and based on HP-filtered data.

Dynare also plots the impulse responses to the shocks. The interpretation is as of any other IRFs. Perhaps the only thing to keep in mind when looking at these results, is that Dynare refers to k_{t+1} as k_t , and hence produced impulse response for k_t is really the impulse response for k_{t+1} .

2 The Models

As a practice and study aid, this guide will present a few examples outlined below:

1. A deterministic Neoclassical growth model with a temporary TFP shock
2. A deterministic Neoclassical growth model with a permanent TFP shock
3. A stochastic Neoclassical growth model
4. A Real Business Cycles model
5. An Open Economy model

2.1 The deterministic neoclassical growth model

A social planner seeks to maximize:

$$\max_{\{c_t, n_t, i_t\}_{t=0}^{\infty}} U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \quad (1)$$

$$s.t. : y_t = z_t k_t^\alpha n_t^{1-\alpha} \quad (2)$$

$$: y_t \geq c_t + i_t \quad (3)$$

$$: k_{t+1} = (1 - \delta)k_t + i_t \quad (4)$$

$$: k_0 > 0 \quad (5)$$

plus a transversality and an Inada condition, as well as a few non-negativity constraints (purposely omitted). Moreover, since we have assumed perfect foresight, the sequence $\{z_t\}_{t=0}^{\infty}$ is predetermined and must also be specified.

Substituting out investment, the lagrangean of this problem would be:

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \lambda_t \left[z_t k_t^\alpha n_t^{1-\alpha} - c_t - k_{t+1} + (1 - \delta)k_t \right] \right\} \quad (6)$$

The first order conditions of the above problem are:

$$(c_t) : c_t^{-\gamma} = \lambda_t \quad (7)$$

$$(k_{t+1}) : \beta E_t \lambda_{t+1} \left[\alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + 1 - \delta \right] = \lambda_t \quad (8)$$

$$(\lambda_t) : A_t k_t^\alpha n_t^{1-\alpha} = c_t + k_{t+1} - (1 - \delta)k_t \quad (9)$$

Note that since in this model the representative agent does not value leisure, it is safe to assume that in equilibrium labor will be supplied inelastically ($\hat{n}_t = 1, \forall t$). Replacing this

above yields:

$$(k_{t+1}) : \beta E_t \lambda_{t+1} \left[\alpha z_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta \right] = \lambda_t \quad (10)$$

$$(\lambda_t) : z_t k_t^\alpha = c_t + k_{t+1} - (1 - \delta)k_t \quad (11)$$

Combining (7) and (10) yields the Euler equation:

$$\beta E_t c_{t+1}^{-\gamma} \left[\alpha z_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta \right] = c_t^{-\gamma} \quad (12)$$

In turn we have 2 endogenous unknowns (c_t, k_{t+1}) and 2 equations (11) and (12).

2.1.1 Calibration

Assuming the model seeks to fit U.S. quarterly data, I will use some standard parameter values to calibrate it³.

Calibration

Parameter	Description	Value	Target/Source
α	Capital share of output	0.33	NIPA Accounts
β	Discount Factor	0.99	Steady state r=4%
δ	Depreciation rate	0.025	10% per year
γ	Utility parameter	0.5	Relative Risk Aversion

2.1.2 Solving for the analytical steady state

At the steady state, $c_t = c_{t+1} = \bar{c}, \forall t$. In turn, from (12) we obtain:

$$\begin{aligned} \bar{c}^{-\gamma} &= \beta \bar{c}^{-\gamma} \left[\alpha \frac{\bar{k}^\alpha}{\bar{k}} + 1 - \delta \right] \\ \Rightarrow 1 &= \beta \left[\alpha \frac{\bar{k}^\alpha}{\bar{k}} + 1 - \delta \right] \\ \Rightarrow \bar{k} &= \left[\left(\frac{1}{\beta} + \delta - 1 \right) \frac{1}{\alpha} \right]^{\frac{1}{\alpha-1}} \end{aligned}$$

which pins down the steady state value of capital.

³See notes on Econ 505 for a quick primer on calibration.

Combining (2) + (3) + (4) yields:

$$\begin{aligned}\bar{A}\bar{k}^\alpha &= \bar{c} + \bar{k} - (1 - \delta)\bar{k} \\ \Rightarrow \bar{k}^\alpha &= \bar{c} + \bar{k} - \bar{k} + \delta\bar{k} \\ \Rightarrow \bar{c} &= \bar{k}^\alpha - \delta\bar{k}\end{aligned}$$

which pins down the steady state level of consumption. Last, the steady state value for the technology shock $\bar{z} = 1$.

The mod-files associated with this model are:

Mod Files

Filename	Description
neo_growth.mod	Deterministic neoclassical model (in levels) with a temporary shock
neo_growth_log.mod	Deterministic neoclassical model (in logs) with a temporary shock
neo_growth2.mod	Deterministic neoclassical model (in levels) with a permanent shock

2.2 The stochastic neoclassical growth model

A social planner seeks to maximize:

$$\max_{\{c_t, n_t, i_t\}_{t=0}^{\infty}} U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \quad (13)$$

$$\text{s.t.} : y_t = z_t k_t^\alpha n_t^{1-\alpha} \quad (14)$$

$$: y_t \geq c_t + i_t \quad (15)$$

$$: k_{t+1} = (1 - \delta)k_t + i_t \quad (16)$$

$$: \ln z_t = \rho \ln z_{t-1} + \epsilon_t \quad (17)$$

$$: k_0 > 0 \quad (18)$$

with $\epsilon_t \sim N(\mu, \sigma)$. Also, note that the above optimization must satisfy a transversality condition, an Inada condition as well as a few non-negativity constraints (purposely omitted).

Substituting out investment, the lagrangean of this problem would be:

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \lambda_t \left[z_t k_t^\alpha n_t^{1-\alpha} - c_t - k_{t+1} + (1 - \delta)k_t \right] \right\} \quad (19)$$

Assuming labor will be inelastically supplied ($\hat{n}_t = 1, \forall t$) in equilibrium, the first order conditions of the above problem are:

$$(c_t) : c_t^{-\gamma} = \lambda_t \quad (20)$$

$$(k_{t+1}) : \beta E_t \lambda_{t+1} \left[\alpha z_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta \right] = \lambda_t \quad (21)$$

$$(\lambda_t) : z_t k_t^\alpha = c_t + k_{t+1} - (1 - \delta)k_t \quad (22)$$

In turn, the set of equations characterizing the equilibrium are:

$$c_t^{-\gamma} = \beta E_t c_{t+1}^{-\gamma} \left[\alpha z_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta \right] \quad (23)$$

$$z_t k_t^\alpha = c_t + k_{t+1} - (1 - \delta)k_t \quad (24)$$

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t \quad (25)$$

We then have 3 endogenous unknowns every period (c_t, k_{t+1}, z_t) and 3 non-linear difference equations to solve for them. In addition, we also require the solution to satisfy the transversality condition: $\lim_{t \rightarrow \infty} \beta^t c_t^{-\gamma} k_{t+1} = 0$

2.2.1 Calibration

Assuming the model seeks to fit US quarterly data, I use some standard parameter values to calibrate it.

Calibration

Parameter	Description	Value
α	Capital share of output	0.33
β	Discount Factor	0.99
δ	Depreciation rate	0.025
γ	Utility parameter	0.5
σ	Standard deviation of TFP shock	0.01
ρ	Autoregressive coefficient on TFP shock	0.95

2.2.2 Solving for the analytical steady state

At the steady state, $c_t = c_{t+1} = \bar{c}, \forall t$. In turn:

$$\begin{aligned}\bar{z} &= 1 \\ \bar{c} &= \beta \bar{c} \left[\alpha \frac{\bar{k}^\alpha}{\bar{k}} + 1 - \delta \right] \\ \Rightarrow \bar{k} &= \left[\left(\frac{1}{\beta} + \delta - 1 \right) \frac{1}{\alpha} \right]^{\frac{1}{\alpha-1}}\end{aligned}$$

which pins down the steady state value of capital. Moreover:

$$\begin{aligned}\bar{A}\bar{k}^\alpha &= \bar{c} + \bar{k} - (1 - \delta)\bar{k} \\ \Rightarrow \bar{k}^\alpha &= \bar{c} + \bar{k} - \bar{k} + \delta\bar{k} \\ \Rightarrow \bar{c} &= \bar{k}^\alpha - \delta\bar{k}\end{aligned}$$

which pins down the steady state level of consumption.

The mod-file associated with this model is:

Mod Files

Filename	Description
neo_growth_log.mod	Deterministic neoclassical model (in logs) with a temporary shock

2.3 The Real Business Cycle model

Two variations of a business cycle model will be introduced in this section: a *classic* version, as well as one with correlated aggregate shocks. Presenting more than one alternative allows us to understand the way in which Dynare handles different model variations. The core model is described below.

2.3.1 A traditional RBC model

Consider an economy with infinitely lived households and a large number of firms producing an homogenous good. This good can either be consumed or invested. Firms own the capital stock and hire labor services from the households, who are also the firms' owners. Every period three perfectly competitive markets open: the markets for consumption goods, labor services and financial capital. A central planner seeks to maximize a representative agent's lifetime utility by choosing consumption, labor effort and investment. Its optimization problem is represented by:

$$\begin{aligned} \max_{\{c_t, n_t, i_t\}_{t=0}^{\infty}} U &= \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - \psi \frac{h_t^{1+\gamma}}{1+\gamma} \right\} \\ \text{s.t.} &: y_t = c_t + i_t \\ &: y_t = e^{z_t} k_t^\alpha n_t^{1-\alpha} \\ &: k_{t+1} = (1 - \delta)k_t + i_t \\ &: z_t = \rho z_{t-1} + \epsilon_t \\ &: n_t + l_t = 1 \\ &: k_0 > 0 \end{aligned}$$

where $0 < \beta < 1$ is the discount factor, c_t consumption and n_t is the labor effort in period t . Moreover, z_t represents a stochastic shock to technology that displays persistence across time with $\epsilon_t \sim N(0, 1)$. Last we assume that $0 < \delta < 1$, $\rho < 1$ and $0 < \alpha < 1$.

The problem's first order conditions for optimality are:

$$\begin{aligned} c_t \psi h_t^{1+\gamma} &= (1 - \alpha) y_t \\ 1 &= \beta \left[\frac{c_t}{c_{t+1}} \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right] \\ y_t &= e^{z_t} k_t^\alpha n_t^{1-\alpha} \\ k_{t+1} &= y_t - c_t + (1 - \delta)k_t \\ z_t &= \rho z_{t-1} + \epsilon_t \end{aligned}$$

which implies that every period we have a system of 5 equations to solve for 5 endogenous variables: y_t, c_t, n_t, k_{t+1} and z_t .

2.3.2 An RBC model with correlated shocks

Assume now there exist a second stochastic disturbance hitting the above economy. In particular, following Juillard and Villemot (2009), I assume that the economy is subject to an investment shock such that:

$$k_{t+1} = e^{\omega_t} i_t + (1 - \delta)k_t$$

Intuitively this shock affects the rate at which new investment can be transformed into productive capital. Further, assume this second disturbance is correlated with the original technology process $\{z_t\}$ in the following manner:

$$\begin{pmatrix} z_t \\ \omega_t \end{pmatrix} = \begin{pmatrix} \rho & \tau \\ \tau & \rho \end{pmatrix} \begin{pmatrix} z_{t-1} \\ \omega_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t \\ v_t \end{pmatrix}$$

where $|\rho + \tau| < 1$ and $|\rho - \tau| < 1$ to guarantee stationarity.

The first order conditions are:

$$\begin{aligned} c_t \psi h_t^{1+\gamma} &= (1 - \alpha)y_t \\ 1 &= \beta \left[\left(\frac{e^{\omega_t} c_t}{e^{\omega_{t+1}} c_{t+1}} \right) \left(\alpha e^{\omega_{t+1}} \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right] \\ y_t &= e^{z_t} k_t^\alpha n_t^{1-\alpha} \\ k_{t+1} &= (y_t - c_t)e^{\omega_t} + (1 - \delta)k_t \\ z_t &= \rho z_{t-1} + \epsilon_t \\ \omega_t &= \rho \omega_{t-1} + v_t \end{aligned}$$

which implies that every period we have a system of 6 equations to solve for 6 endogenous variables: $y_t, c_t, n_t, k_{t+1}, \omega_t$ and z_t .

2.3.3 Calibration

Assuming the model seeks to fit US data, the chosen parameter values are:

Parameter	Description	Value
α	Capital share of output	0.36
β	Discount Factor	0.99
δ	Depreciation rate	0.025
γ	Labor supply elasticity	0
ψ	Disutility of Labor	2.95
ρ	Autoregressive coefficient on shocks	0.95
τ	Cross-persistence on shocks	0.025
σ	Standard deviation of shocks	0.009

An interesting feature of this model is that it incorporates correlated disturbances. Note that we have:

$$\begin{aligned}
 E(\epsilon_t) &= 0 \\
 E(v_t) &= 0 \\
 E(\epsilon_t, \epsilon_s) &= \begin{cases} \sigma_\epsilon^2 & \text{if } t = s \\ 0 & \text{if } t \neq s. \end{cases} \\
 E(v_t, v_s) &= \begin{cases} \sigma_v^2 & \text{if } t = s \\ 0 & \text{if } t \neq s. \end{cases} \\
 E(\epsilon_t, v_s) &= \begin{cases} \varphi \sigma_\epsilon^2 \sigma_v^2 & \text{if } t = s \\ 0 & \text{if } t \neq s. \end{cases}
 \end{aligned}$$

where φ is the conditional correlation of the shocks. While this is not a parameter of the system and strictly speaking would not need to be declared in the mod file, it will turn out to be convenient to do so.

To specify the innovations and their matrix of variance-covariance, one does this using the *shocks*; and *end*; block. For example in this case we would have:

```
shocks;
var e = 0.009 ^ 2;
var nu = 0.009 ^ 2;
var e, nu = VARPFI * 0.009 * 0.009;
end;
```

where I have set $\varphi = 0.1$. Note that it is possible to shut down either shock by assigning it a zero variance.

The mod-file associated with these models are:

Mod Files

Filename	Description
rbc1_log.mod	<i>Traditional</i> RBC model (in logs)
rbc2_log.mod	RBC model (in logs) with correlated shocks

2.4 International Business Cycle Model

Schmitt-Grohe & Uribe (2003) show us how to "close" a small open economy model. In other words, how to induce stationarity into an open economy framework that would otherwise have a random walk component. To resolve this issue they propose five specifications:

1. A model with an endogenous discount factor
2. A model with a debt-elastic interest rate premium
3. A model with convex portfolio adjustment costs
4. A model with complete asset markets
5. A model without stationary-inducing features

I will refer to the second type of model to provide the Dynare specification.

Consider a small open economy populated by a large number of identical households with lifetime preferences denoted by:

$$\max_{\{c_t, h_t\}} U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{[c_t - \omega^{-1} h_t^\omega]^{1-\gamma} - 1}{1-\gamma} \right)$$

with $0 < \beta < 1$ is a constant discount factor. Technology is given by:

$$\begin{aligned} y_t &= A_t k_t^\alpha h_t^{1-\alpha} \\ \text{s.t.} \quad & \ln A_t = \rho \ln A_{t-1} + \epsilon_t \end{aligned}$$

where $\{A_t\}$ is an exogenous, stochastic productivity shock. The stock of capital evolves according to:

$$k_{t+1} = i_t + (1 - \delta)k_t$$

where $\delta \in (0, 1)$ denotes the rate of depreciation of physical capital. Moreover, stationarity will be induced by assuming that the interest faced by domestic agents r_t is increasing in the aggregate level of foreign debt, which is denoted by \hat{d}_t . Specifically, r_t is given by:

$$r_t = r^w + p(\hat{d}_t)$$

where r^w denotes the world interest rate and $p(\cdot)$ is a country-specific interest rate premium which is modeled as:

$$p(d_t) = \psi(e^{d_t - \bar{d}} - 1)$$

where ψ and \bar{d} are known parameters. Lastly, the evolution of the foreign debt follows:

$$d_t = (1 + r_{t-1})d_{t-1} - y_t + c_t + i_t + \phi(k_{t+1} - k_t)$$

The system of equation that characterize the model's solution is:

$$\begin{aligned}
d_t &= (1 + r_{t-1})d_{t-1} - y_t + c_t + i_t + \phi(k_{t+1} - k_t) \\
r_t &= r^w + p(\hat{d}_t) \\
y_t &= A_t k_t^\alpha h_t^{1-\alpha} \\
\ln A_t &= \rho \ln A_{t-1} + \epsilon_t \\
p(d_t) &= \psi(e^{d_t - \bar{d}} - 1) \\
k_{t+1} &= i_t + (1 - \delta)k_t \\
(c_t - \omega^{-1}h_t^\omega)^{-\gamma} &= \beta(1 + r_t)E_t(c_{t+1} - \omega^{-1}h_{t+1}^\omega)^{-\gamma} \\
(c_t - \omega^{-1}h_t^\omega)^{-\gamma}h^\omega &= (c_t - \omega^{-1}h_t^\omega)^{-\gamma}(1 - \alpha)y_t \\
(c_t - \omega^{-1}h_t^\omega)^{-\gamma}[1 + \phi(k_{t+1} - k_t)] &= \beta E_t(c_{t+1} - \omega^{-1}h_{t+1}^\omega)^{-\gamma} \left[\alpha \left(\frac{y_{t+1}}{k_{t+1}} \right) + 1 - \delta + \phi(i_{t+1} - \delta k_{t+1}) \right]
\end{aligned}$$

In short, 9 equations for 9 endogenous variables: $c_t, d_t, h_t, y_t, i_t, k_{t+1}, A_t, r_t$ and $p(d_t)$.

The parametrization is as follows:

Calibration

Parameter	Description	Value
α	Capital share of output	0.32
ρ	Autoregressive coefficient on TFP shock	0.42
ψ	Coefficient on i-rate premium	0.000742
ϕ	Coefficient on foreign debt law of motion	0.028
γ	Utility parameter	2
r^w	World Interest rate	0.04
ω	Utility parameter	1.455
δ	Depreciation rate	0.1
σ	Standard deviation of TFP shock	0.0129
β	Discount Factor	$\frac{1}{(1+r^w)}$
\bar{d}	Steady state level of foreign debt	0.7442

Mod Files

Filename	Description
noem.mod	Small Open Economy model (in logs)