Notes on Monetary Theory
(a pathetically short introduction)

Diego Vilán *

Spring 2011

Broadly speaking, monetary economics investigates the relationship between real aggregate economic variables (output, investment, employment) and nominal ones (money supply, nominal interests and exchange rates). The focus on monetary economics emphasizes price-level determination, inflation and the role of monetary policy.

Seminal work of Robert Lucas (1972) provided the first theoretical foundation for models of economic fluctuations in which money was the fundamental driving factor behind movements in real output. The rise of real business cycle theory during the 1980s and 1990s, building on the contributions of Kydland and Prescott (1982) (who focused explicitly on non-monentary factors as the driving forces behind business cycles), would set the methodological gold standard as researchers sought to incorporate monetary factors into dynamic general equilibrium models.

Central to the discussion of how nominal variables affect real ones are the notions of money, its functions, as well as the speed at which prices may fluctuate to clear markets. In the classical approach prices and wages are assumed to adjust quickly so that markets are almost always in equilibrium. In view of the above, classical economists argue that business cycles represent's the economy's best response to disturbances such as productivity shocks.

In contrast, Keynesians are less optimistic about the ability of free-markets to respond quickly and efficiently to exogenous disturbances. Wage and price rigidity implies that the economy can be away from its general equilibrium for significant periods of time. Thus a recession is not an optimal response of the free market to a shocks; rather, it is a disequilibrium situation in which high unemployment reflects and excess supply of labor.

The present note seeks to introduce two alternative set of frameworks useful for the study of price dynamics and monetary theory. One model which incorporates money in the utility function under the classical vision of flexible prices and instantaneous adjustment. The other one incorporates imperfect competition and constraints on the firm's ability to change prices.

*****DISCLAIMER**: I wrote these notes as a study aid for myself. They are work in progress and could be incomplete, inaccurate and even somewhat incorrect. Keep that in mind should you decide to use them. Comments and suggestions welcomed!

Model 1: The Classical Monetary model

Most of the classical models do not incorporate monetary assets explicitly in their analysis. Under these frameworks, the only role played by money is to serve as a numeraire (ie: a unit of account in which prices, wages and securities' payoffs are stated). Economies with such characteristic are often referred to as *cashless economies*.

This section presents a simple model of a classical monetary economy, featuring perfect competition and flexible prices in all markets, where an explicit role for money (beyond that of serving as a unit of account) is introduced. In particular, real balances are assumed to generate utility to households, and the implications for monetary policy will depend on the assumption of the used utility function. The main properties of the model are summarized below:

Key features:

- No physical capital. Labor as sole factor of production.
- One consumption good
- Money as an additional good (included in the agent's utility function[1])
- Perfect competition and fully flexible prices in all markets

The Firm's problem:

$$\max_{\{N_t\}_{t=0}^{\infty}} \pi_t = p_t Y_t - w_t N_t \quad (1)$$

$$s.t. \quad : \quad Y_t = A_t N_t^{1-\alpha} \quad (2)$$

Assuming an interior solution, the FOC is:

$$[N_t] \quad : \quad w_t = (1-\alpha) p_t A_t N_t^{-\alpha} \quad (3)$$

$$: \quad \left(\frac{w_t}{p_t}\right) = (1-\alpha) A_t N_t^{-\alpha} \text{(demand for labor)} \quad (4)$$

[1]Period utility is assumed to be concave and increasing in real balances.

The Household's problem:

$$\begin{aligned} & \max_{\{c_t, N_t, m_t, b_t\}_{t=0}^{\infty}} E_0 \sum \beta^t u(c_t, N_t, m_t) && (5) \\ s.t. \quad : \quad & b_{t-1} + m_{t-1} + w_t N_t - T_t \geq p_t c_t + q_t b_t + m_t \\ : \quad & \lim_{T \to \infty} E_t \{b_T\} \geq 0 \end{aligned}$$

where:

(i) $m_t = \frac{M_t}{p_t}$ are the agent's real balances.

(ii) p_t is the price of the consumption good and w_t the nominal wage.

(iii) b_t represents the quantity of one-period, nominally discount bonds purchased at time t and maturing at $t+1$.

(iv) q_t is the price at time t of a bond that pays one unit of money at $t+1$.

Assuming there is no long-term inflation (case in which we would need to make the model stationary) one could write the above problem in recursive format:

$$\begin{aligned} V(m_{-1}, b_{-1}, S) &= \max_{c,n,M,b} \left\{ u(c, N, m) + \beta E V(m, b, S') \right\} && (6) \\ s.t. \quad &: \quad b_{-1} + M_{-1} + wN - T \geq pc + qb + M \\ &: \quad M = (1-\psi)M + \phi M_{-1} + \varepsilon \text{ (LOM of Money Supply)} \end{aligned}$$

where S_t refers to the economy's aggregate states:

$$\begin{aligned} M_{t-1} &: \text{ Aggregate Supply of Money at } t-1 \\ A_t &: \text{ Technology shock at time } t \\ \Delta M_t &: \text{ Change in the Money Supply at time } t \end{aligned}$$

Assuming an interior solution, the FOCs are:

$$[N_t] : u_{2t} + \lambda_t w_t = 0 \tag{7}$$

$$[c_t] : u_{1t} - \lambda_t p_t = 0 \tag{8}$$

$$[M_t] : u_{3t} \frac{1}{p_t} + \beta E V_{nt} = \lambda_t \tag{9}$$

$$[b_t] : \beta E V_{bt} = \lambda_t q_t \tag{10}$$

Envelop conditions:

$$V_{nt-1} = \lambda_t \tag{11}$$
$$V_{bt-1} = \lambda_t \tag{12}$$

Combining (7) and (8):

$$\underbrace{\frac{w_t}{p_t}}_{\substack{\text{Benefit from an}\\ \text{extra unit of work}\\ \text{(real wage)}}} \underbrace{u_{1t}}_{\substack{\text{Mg utility}\\ \text{of } c_t}} = \underbrace{-u_{2t}}_{\substack{\text{Mg disutility of an}\\ \text{extra unit of labor}}} \quad \forall t \tag{13}$$

$$(\text{marginal benefit} = \text{marginal cost})$$

Intratemporal optimality requires that the marginal gain from consumptions equals the marginal desutility of labor at every point in time. Rearranging:

$$\Rightarrow \frac{w_t}{p_t} = \frac{-u_{2t}}{u_{1t}} \tag{14}$$

Combining (8) and (10) yields:

$$\begin{aligned} u_{1t} &= \beta E_t \left(\frac{p_t}{q_t p_{t+1}} \right) u_{1t+1} \\ &= \beta \left\{ \frac{p_t}{q_t} E_t \left(\frac{1}{p_{t+1}} \right) \right\} u_{1t+1} \end{aligned} \tag{15}$$

Intertemporal optimality requires:

$$u_{1t} = u_{3t} + \beta \left(\frac{p_t}{p_{t+1}} \right) u_{1t+1} \tag{16}$$

Now considering the following utility function[2]:

$$u\left(c_t, N_t, \frac{M_t}{p_t}\right) = \frac{c_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1-\psi}}{1-\psi} + \frac{(M_t/p_t)^{1-\nu}}{1-\nu}$$

the associated lagrangean would be:

[2]For a derivation with non-separable utility see Jordi Gali Chapter 2.

$$L=\sum_{t=0}^{\infty}\beta^t\left\{\frac{c_t^{1-\gamma}}{1-\gamma}-\frac{N_t^{1-\psi}}{1-\psi}+\frac{(M_t/p_t)^{1-\nu}}{1-\nu}+\lambda_t\left[b_{t-1}+M_{t-1}+w_tN_t-T_t-p_tc_t-q_tb_t-M_t\right]\right\}$$

re-write the FOCs as:

$$c_t^{\gamma}N_t^{\psi}=\frac{w_t}{p_t} \tag{17}$$

$$c_t^{-\gamma}=\beta E_t\left(\frac{p_t}{q_tp_{t+1}}\right)c_{t+1}^{-\gamma} \tag{18}$$

$$(1-q_t)c_t^{-\gamma}=\left(\frac{M_t}{p_t}\right)^{-\nu} \tag{19}$$

where (17) is the intratemporal optimality condition for labor, (18) the Euler equation for consumption and (19) is the intratemporal condition for real balances. Note that from (19) we can derive the agent's demand for real balances, which is an increasing function of consumption and inversely related to the interest rate:

$$\begin{aligned}\frac{M_t}{p_t} &= \left[(1-q_t)c_t^{\gamma}\right]^{\frac{1}{\nu}} \\ &= \left[(1-e^{-i_t})c_t^{\gamma}\right]^{\frac{1}{\nu}}\end{aligned} \tag{20}$$

Since we have one choice variable for the firm N_t (demand for labor) and four for the household: c_t, M_t, B_t, N_t (labor supply), we have 5 unknowns and thus need 5 equations. To complete the system (17)-(19) we use:

$$\left(\frac{w_t}{p_t}\right)=A_t(1-\alpha)N_t^{-\alpha} \tag{21}$$

$$Y_t=A_tN_t^{1-\alpha} \tag{22}$$

where (21) is the firm's demand for labor and (22) the production technology.

We seek to linearize the above system. Begin by equation (17):
1) take logs:

$$-\gamma\ln c_t=\ln w_t-\ln p_t+\psi\ln N_t$$

2) evaluate at steady state:

$$-\gamma\ln\bar{c}=\ln\bar{w}-\ln\bar{p}+\psi\ln\bar{N}$$

3) perform linear approximation

$$
\begin{aligned}
-\gamma \ln \bar{c} - \gamma \hat{c}_t &= \ln \bar{w} + \hat{w}_t - \ln \bar{p} - \hat{p}_t + \psi \ln \bar{N} + \psi \hat{N}_t \\
&\Rightarrow \hat{w}_t - \hat{p}_t = \gamma \hat{c}_t + \psi \hat{N}_t
\end{aligned}
$$

which yields an expression for the real wage.

Equation (18):
1) take logs:

$$
-\gamma \ln c_t = \ln \beta + E_t \{\ln p_t - \ln q_t - \ln p_{t+1} - \gamma \ln c_{t+1}\}
$$

2) evaluate at steady state:

$$
\begin{aligned}
-\gamma \ln \bar{c} &= \ln \beta + E_t \{\ln \bar{p} + \ln \bar{q} - \ln \bar{p} - \gamma \ln \bar{c}\} \\
&\Rightarrow \ln \beta = \ln \bar{q}
\end{aligned}
$$

3) perform linear approximation:

$$
\begin{aligned}
-\gamma \ln \bar{c} - \gamma \hat{c}_t &= \ln \beta + E_t \{\ln \bar{p} + \hat{p}_t - \ln \bar{q} - \hat{q}_t - \ln \bar{p} - \hat{p}_{t+1} - \gamma \ln \bar{c} - \gamma \hat{c}_{t+1}\} \\
&\Rightarrow -\gamma \hat{c}_t = E_t \{-\hat{q}_t + \hat{p}_t - \hat{p}_{t+1} - \gamma \hat{c}_{t+1}\} \\
&\Rightarrow \hat{c}_t = E_t \hat{c}_{t+1} + \frac{1}{\gamma}[E_t \hat{p}_{t+1} - \hat{p}_t + \hat{q}_t] \qquad (23)
\end{aligned}
$$

where:

$$
\begin{aligned}
E_t \hat{p}_{t+1} - \hat{p}_t &= E_t(\ln p_{t+1} - \ln \bar{p}) - \ln p_t + \ln \bar{p} \\
&= E_t \ln p_{t+1} - \ln p_t \\
&= E_t \pi_{t+1}
\end{aligned}
$$

and

$$
\begin{aligned}
q_t &= \frac{1}{1+i_t} \\
\hat{q}_t &= \ln\left(\frac{1}{1+i_t}\right) - \ln\left(\frac{1}{1+\bar{\imath}}\right) \\
&= -\ln(1+i_t) + \ln(1+\bar{\imath}) \\
&\approx -(i_t - \bar{\imath})
\end{aligned}
$$

where i_t is the nominal interest rate[3]. Combining all of the above yields:

[3]Note that it is common to see $i_t \equiv -\ln q_t$. This is because:

$$\hat{c}_t = E_t\hat{c}_{t+1} + \frac{1}{\gamma}[i_t - E_t\pi_{t+1} - \bar{\imath}]$$

which implies that the expected real interest $(i_t - E_t\pi_{t+1})$ matters for the agent's intertemporal allocations.

Forgetting about the constant $\bar{\imath}$ and knowing that in equilibrium consumption will equal production $(y_t = c_t)$, the expression above becomes:

$$E_t y_{t+1} - y_t = \frac{1}{\gamma}[i_t - E_t\pi_{t+1}] \tag{24}$$

Hence if today the expected inflation goes up (and consequently so does the real interest rate), agents in this model expect output to go up as well. In terms of the Euler equation, agents will consume less today, save and invest more, and seek to increase future consumption.

Equation (19):
1) rewrite in exponential form:

$$(1 - e^{\ln q_t})e^{-\gamma \ln c_t} = e^{-\nu \ln M_t + \nu \ln p_t}$$

2) approximate at steady state:

$$\begin{aligned}
-e^{\ln\bar{q}}e^{-\gamma\bar{c}}\hat{q}_t - \gamma(1 - e^{\ln\bar{q}})e^{-\gamma\ln\bar{c}}\hat{c}_t &= -\nu e^{-\nu\ln\bar{M}+\nu\ln\bar{p}}\hat{M}_t + e^{-\nu\ln\bar{M}} + \nu\ln\bar{p}\hat{p}_t \\
&\Rightarrow -\hat{q}_t - \gamma\hat{c}_t = -\nu\hat{M}_t + \nu\hat{p}_t \\
&\Rightarrow \hat{M}_t - \hat{p}_t = \frac{\gamma}{\nu}\hat{c}_t - \frac{1}{\nu}(i_t - \bar{\imath})
\end{aligned}$$

The above is the money demand function, where real balances $(\hat{M}_t - \hat{p}_t)$ are a function of consumption and the interest rate. Given that in equilibrium consumption is equal to output we have that:

$$\hat{M}_t - \hat{p}_t = \frac{\gamma}{\nu}\hat{y}_t - \frac{1}{\nu}(i_t - \bar{\imath})$$

Equation (21):

$$\begin{aligned}
q_t &= \frac{1}{1+i_t} \\
\ln q_t &= \ln(1) - \ln(1+i_t) \\
&= -\ln(1+i_t) \\
&\simeq -i_t \\
&\Rightarrow i_t \simeq -\ln q_t
\end{aligned}$$

1) take logs:

$$\ln w_t - \ln p_t = \ln A_t + \ln(1-\alpha) - \alpha \ln N_t$$

2) evaluate at steady state:

$$\ln \bar{w} - \ln \bar{p} = \ln(1-\alpha) + \ln \bar{A} - \alpha \ln \bar{N}$$

3) approximate at steady state:

$$\begin{aligned} \ln \bar{w} + \hat{w}_t - \ln \bar{p} - \hat{p}_t &= \ln(1-\alpha) + \ln \bar{A} + \hat{A}_t - \alpha \ln \bar{N} - \alpha \hat{N}_t \\ &\Rightarrow \hat{w}_t - \hat{p}_t = A_t - \alpha \hat{N}_t \end{aligned} \tag{25}$$

Equation (22):
1) take logs:

$$\ln Y_t = \ln A_t + (1-\alpha) \ln N_t$$

2) evaluate at steady state:

$$\ln \bar{Y} = \ln \bar{A} + (1-\alpha) \ln \bar{N}$$

3) approximate at steady state:

$$\begin{aligned} \ln \bar{Y} + \hat{y}_t &= \ln \bar{A} + \hat{A}_t + (1-\alpha) \ln \bar{N} + (1-\alpha) \hat{N}_t \\ &\Rightarrow \hat{y}_t = \hat{A}_t + (1-\alpha) \hat{N}_t \end{aligned} \tag{26}$$

To sum up, in equilibrium the linearized system is as follows:
(a) From the HH:

$$\text{Labor supply: } \hat{w}_t - \hat{p}_t = \gamma \hat{y}_t + \psi \hat{N}_t \tag{27}$$

$$\text{Intertemporal: } E_t y_{t+1} - y_t = \frac{1}{\gamma}[i_t - E_t \pi_{t+1}] \tag{28}$$

$$\text{Money demand: } \hat{M}_t - \hat{p}_t = \frac{\gamma}{\nu} \hat{y}_t - \frac{1}{\nu}(i_t - \bar{\imath})^4 \tag{29}$$

(b) From the Firm:

$$\text{Labor demand: } \hat{w}_t - \hat{p}_t = A_t - \alpha \hat{N}_t \tag{30}$$

[4] Note that if we ignore the constant and focus on the particular case when $\gamma = \nu$ (which implies a unit elasticity with respect to consumption) we obtain the conventional, linear demand for real balances: Letting $\eta = \frac{1}{\nu}$, yields: $\hat{M}_t - \hat{p}_t = \hat{y}_t - \eta i_t$

$\eta \geq 0$ is often referred to as semi-elasticity of money demand.

$$\text{Production: } \hat{y}_t = \hat{A}_t + (1-\alpha)\hat{N}_t \tag{31}$$

With the above we can pin down the model's real variables.

In equilibrium (27) should equal (30):

$$\hat{A}_t - \alpha \hat{N}_t = \gamma \hat{y}_t + \psi \hat{N}_t$$

using (31), yields:

$$\begin{aligned}
\hat{A}_t - \alpha \hat{N}_t &= \gamma[\hat{A}_t + (1-\alpha)\hat{N}_t] + \psi \hat{N}_t \\
&= \gamma \hat{A}_t + \gamma(1-\alpha)\hat{N}_t + \psi \hat{N}_t \\
&\Rightarrow \hat{A}_t - \gamma \hat{A}_t = \gamma(1-\alpha)\hat{N}_t + \psi \hat{N}_t + \alpha \hat{N}_t \\
&\Rightarrow A_t(1-\gamma) = \hat{N}_t[\gamma(1-\alpha) + \psi + \alpha] \\
&\Rightarrow \hat{N}_t = \frac{\hat{A}_t(1-\gamma)}{[\gamma(1-\alpha) + \psi + \alpha]} \qquad (32)
\end{aligned}$$

which implies that equilibrium labor demand will be entirely determined by productivity. In turn, so will be output:

$$\Rightarrow \hat{y}_t = \left[\frac{(1+\psi)}{(1-\alpha)\gamma + \alpha + \psi}\right] \hat{A}_t \tag{33}$$

Moreover, to pin down the real interest rate we combine the Fisher equation ($r_t = i_t - E_t \pi_{t+1}$) with (28) and ignore the constant term. This yields:

$$r_t = \left[\frac{\gamma(1+\psi)}{(1-\alpha)\gamma + \alpha + \psi}\right] E_t[\hat{A}_{t+1} - \hat{A}_t] \tag{34}$$

which implies that the real interest rate depends only on the expected productivity difference.

Overall this suggests that the equilibrium dynamics of all real variables (output, employment, and the real interest rate) are determined independently of monetary policy. In this simple model, output and employment fluctuate in response to variations in technology, which is assumed to be the only real driving force. This property is often referred to as neutrality of money.

In particular, we have that:

1. Output always rises in the face of a productivity increase. The same is true for the real wage.

2. The direction of labor supply depends on the parameter γ, which captures the strength of the wealth effect of labor supply). When $\gamma < 1$ the substitution effect on labor supply resulting from a higher wage dominates the negative effect caused by a smaller marginal utility of consumption, leading to an increase in employment. The converse is true when $\gamma > 1$. If the utility of consumption is logarithmic $(\gamma = 1)$, then labor supply remains unchanged in the face of technology shocks, for substitution and income effects cancel each other.

3. The response of the real interest rate depends on the time series properties of the technological shock. If the current improvement is transitory, so that $E_t \hat{A}_{t+1} < \hat{A}_t$ then the real rate will go down at time t (see equation (34)). If technology is expected to keep improving, then $E_t \hat{A}_{t+1} > \hat{A}_t$ and the real rate will increase with a rise in $\hat{A}_t$.

4. Monetary policy will, however, affect nominal variables. In turn, their equilibrium behavior will not be determined uniquely by real forces. Instead, to pin down the price level (and hence inflation and nominal interest rate) we will need one more equation that specifies how monetary policy is conducted[5].

We study two cases, one in which the central bank uses monetary aggregates as its policy instrument and another one in which the monetary authority targets a particular interest rate.

(1) The nominal interest rate as the policy tool:

(a) Consider first the case of the nominal interest following an exogenous stationary process $\{i_t\}$. Without loss of generality we assume that the process has a mean ω, which is consistent with a steady state with zero inflation and no secular growth. For example, this corresponds to the case when the Central Bank seeks to keep a constant interest rate $i_t = i = \omega$, for all t.

Recall the Fisher equation:

$$\begin{aligned} i_t &= E_t \pi_{t+1} + r_t \\ &\Rightarrow E_t \pi_{t+1} = i_t - r_t \end{aligned} \tag{35}$$

where r_t will be determined independently of monetary policy.

Note that expected inflation will be pinned down by (35), yet actual inflation (and hence the price level) is not. Since there is no other condition that can be used to determine inflation, it follows that any path for the price level that satisfies

$$p_{t+1} = p_t + i_t - r_t + \xi_{t+1} \tag{36}$$

[5] See also comments on optimal monetary policy.

would be consistent with equilibrium, where ζ_{t+1} is a shock not related to fundamentals. Such shocks are often referred to as sunspot shocks. We therefore conclude that an exogenous nominal interest rate leads to price indeterminacy.

(b) Suppose now that the central bank adjusts the nominal interest rate according to a rule that takes into account the inflation rate:

$$i_t = \eta + \phi_\pi \pi_t \tag{37}$$

where $\phi_\pi \geq 0$. Combining (37) with the Fisher equation yields:

$$\begin{aligned} \phi_\pi \pi_t &= E_t \pi_{t+1} + \tilde{r}_t \\ \text{where } \tilde{r}_t &\equiv r_t - \eta \\ &\Rightarrow \pi_t = \frac{1}{\phi_\pi} \left[E_t \pi_{t+1} + \tilde{r}_t \right] \end{aligned} \tag{38}$$

We consider two cases of interest, depending on whether the coefficient of inflation in the above rule is larger or smaller than one. The various coefficient values imply different types of reactions on behalf of the monetary authority:

$$\begin{aligned} \text{Aggresive reaction} &: \quad \phi_\pi > 1 \\ \text{Weak reaction} &: \quad \phi_\pi < 1 \\ \text{No reaction} &: \quad \phi_\pi = 0 \end{aligned}$$

If $\phi_\pi > 1$, equation (38) has a stationary solution. That solution can be found by iterating the equation forward to obtain:

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t \tilde{r}_{t+k} \tag{39}$$

which fully determines inflation (and hence the price level) as a function of the real interest which we've seen is a function of fundamentals. For example, consider the case in which the technology follows a stationary AR(1) process:

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \hat{\varepsilon}_t^A \tag{40}$$

then (34) implies $\tilde{r}_t = -\sigma \left[\frac{1+\psi}{\sigma(1-\alpha)+\psi+\alpha} \right] (1-\rho_A) \hat{A}_t$, which combined with (39) yields the needed expression for inflation equilibrium:

$$\pi_t = \left[\frac{\sigma \left[\frac{1+\psi}{\sigma(1-\alpha)+\psi+\alpha} \right] (1-\rho_A)}{\phi_\pi - \rho_A} \right] \hat{A}_t \tag{41}$$

The above highlights how in this set-up, inflation dynamics will be affected by technological shocks. Note that the central bank, following a rule of the form considered here can influence the degree of inflation in the economy by choosing the size of ϕ_π. The bigger the parameter (ie: the larger away from 1 it is) the smaller will be the impact of the real shock on inflation.

On the other hand, if $\phi_\pi < 1$, the stationary solution to (38) takes the form

$$\pi_{t+1} = \phi_\pi \pi_t - \hat{r}_t + \xi_{t+1} \tag{42}$$

where $\{\xi_t\}$ is, again, an arbitrary sequence of shock, possibly unrelated to fundamentals, satisfying $E_t\{\xi_{t+1}\} = 0$ for all t. Accordingly, any process $\{\pi_t\}$ satisfying (42) is consistent with equilibrium and so as in the case of an exogenous nominal rate, the price level (and hence inflation and the nominal rate) are not determined uniquely. In short, when the interest rate rule implies a weak response of the nominal rate to changes in inflation, the model is indetermined.

All in all, under a interest base rule, the condition for a determinate price level is $\phi_\pi > 1$. This requires the central bank to adjust nominal interest rates more than one-for-one in response to any change in inflation, a property known as the Taylor principle.

(2) Using monetary aggregates as the policy tool:

Now let us consider the case in which the monetary authority chooses to set a specific path for monetary aggregates.

Suppose that the central bank sets an exogenous path for the money supply $\{M_t\}$. Using (35) to eliminate the nominal interest rate in (29), we can derive the following difference equation for the price level:

$$\begin{aligned}
\hat{M}_t - \hat{p}_t &= \frac{\gamma}{\nu}\hat{y}_t - \frac{1}{\nu}\left[E_t\pi_{t+1} + r_t - \bar{\imath}\right] \\
&\Rightarrow \hat{p}_t = \hat{M}_t - \frac{\gamma}{\nu}\hat{y}_t + \frac{1}{\nu}\left[E_t\hat{p}_{t+1} - \hat{p}_t + r_t - \bar{\imath}\right] \\
&\Rightarrow \hat{p}_t\left(1 + \frac{1}{\nu}\right) = \hat{M}_t - \frac{\gamma}{\nu}\hat{y}_t + \frac{1}{\nu}\left[E_t\hat{p}_{t+1} + r_t - \bar{\imath}\right] \\
&\Rightarrow \hat{p}_t\left(\frac{1+\nu}{\nu}\right) = \hat{M}_t - \frac{\gamma}{\nu}\hat{y}_t + \frac{1}{\nu}\left[E_t\hat{p}_{t+1} + r_t - \bar{\imath}\right]
\end{aligned}$$

$$\Rightarrow \hat{p}_t = \left(\frac{1}{1+\nu}\right)E_t\hat{p}_{t+1} + \left(\frac{\nu}{1+\nu}\right)\hat{M}_t + \mu_t \tag{43}$$

where

$$\mu_t = \frac{1}{1+\nu}(r_t - \bar{\imath} - \gamma \hat{y}_t) \tag{44}$$

Assuming $\nu > 0$ and iterating forward yields:

$$\hat{p}_t = \frac{1}{1+\nu} \sum_{k=0}^{\infty} \left(\frac{\nu}{1+\nu}\right)^k E_t \hat{M}_{t+k} + \mu_t' \tag{45}$$

where

$$\mu_t' \equiv \sum_{k=0}^{\infty} \left(\frac{\nu}{1+\nu}\right)^k E_t \mu_{t+k} \tag{46}$$

hence we see how an arbitrary exogenous path for the money supply always determined the price level uniquely. Given the price level as determines above, we can use (29) to solve for the nominal interest rate:

$$i_t = v[\hat{y}_t - (\hat{M}_t - \hat{p}_t)]$$

As an example, consider the case when monetary policy follows an AR(1) process:

$$M_{t+1} = \bar{M}(1-\rho) + \rho M_t + \varepsilon_{t+1} : \rho < 1 \tag{47}$$

which linearized becomes:

$$\hat{M}_{t+1} = \rho \hat{M}_t + \hat{\varepsilon}_{t+1} \tag{48}$$

Forward substitution yields:

$$\begin{aligned} \hat{p}_t &= \left(\frac{v}{1+v}\right) \sum_{k=0}^{\infty} \left(\frac{\rho}{1+v}\right)^k E_t \hat{M}_{t+k} + \mu_t' \\ \Rightarrow \quad \hat{p}_t &= \underbrace{\left(\frac{v}{1+v-\rho}\right)}_{>1} \hat{M}_t + \underbrace{\mu_t''}_{\text{Real output}} \end{aligned} \tag{49}$$

(49) implies that prices should respond more than one-for-one for an exogenous change in money supply. However, this is in sharp contrast to the sluggish response of the price level usually observed in the data.

Conclusion: The baseline model outlined above represents a slight variation of a classic GE framework that generates a motive to hold money by introducing real balances into the agent's utility function. Overall this classical monetary economy is characterized by neutrality of money and efficiency of the equilibrium allocation. In particular, the analysis of the model has shown that while real variables are independent of monetary policy, the latter can have important implications for the behavior of nominal variables and, in particular, of prices. However, the classical monetary framework fails to establish any relationship of monetary policy on real variables, a feature which some have argued is in clear contrast with the existing empirical evidence. In turn, this shortcoming constitutes the main motivation behind the introduction of nominal frictions in the following model.

On optimal monetary policy: If we consider a classic GE model without any money in the agent's utility function, we find that all the conclusions about price level determinacy still apply. However, given that the household's utility is a function of consumption and worked hours only (two real variables that are invariant to monetary policy is conducted) it follows that there is no policy rule that is better (ie: optimal) than any other. Thus in such a model a policy that generates large fluctuations in inflation and other nominal variables is no less desirable that the one that succeeds in stabilizing prices.

By introducing money in the agent's utility function real balances are assumed to yield utility and therefore monetary policy has the capacity of affecting welfare. As such, some policy rules will be preferred to others and the discussion of optimal monetary policy is relevant.

Model 2: The basic New Keynesian model

The sluggish adjustment of nominal wages and prices is central to the Keynesian thought, and investigating the microeconomic foundations of such mechanism is necessary for constructing fully specified models. There are many reasons why this might occur including efficiency wages, long-term contracts and collective bargaining agreements to mention a few. The literature has primarily focused on two cases: imperfect information, and frictions to nominal adjustments.

The Lucas-Phelps imperfect information model:

The first idea corresponds to the Lucas-Phelps imperfect information model. The central idea here is that when a producer observes a change in its product's price, he/she does not know whether it reflects a change in the good's relative price or a change in the aggregate price level. Naturally, a change in the relative price alters the optimal amount to produce, while a change in the aggregate price level leaves optimal production unchanged.

When the price of the producer's good increases, there is some change that the increase reflects a rise in the price level, and some chance that it reflects a rise in the good's relative price. The rational response for the producer is to attribute part of the change to an increase in the price level and part to an increase in the relate price, and therefore to increase output somewhat. This implies that the aggregate supply curve slopes up: when the aggregate price level rises, all producers infer increases in the price of their goods and (not knowing that the increase actually reflects a rise in the general price level) decide to raise their output.

Frictions to nominal adjustments:

At the core of New Keynesian models lies the incomplete adjustment of nominal prices and wages. The Keynesian explanation for the existence of price rigidities rely on two main ideas: (1) most firms actively set the prices of their products (rather than taking them as given) and (2) when firms change prices they must face some cost or friction. In other words, firms are price setters but they are somewhat constrained both by their own product's demand as well as by some type of nominal rigidity.

The present section will describe the key elements of the framework usually referred to as the new Keynesian model. The departure from the assumptions of the classical monetary economy are not trivial. In particular, because firms are now price setters we will find it necessary to develop a framework to think about monopolistic competition which these models will be based on. The models' key features are specified below while some details are left for the Appendix.

Key features:

- Product differentiation
- Imperfect (monopolistic) competition
- Agents consume an assortment of goods
- Nominal rigidity: a) Menu Costs, b) Staggered Price setting

Menu Costs and the Rotemberg Model

An important theory of price stickiness is based on the notion that the very act of changing prices itself entails a cost. The classic example is the case of a restaurant that needs to decide whether or not to print new menus to update some of its prices. The restaurant will only bear the cost of printing new menus, if it expects to generate even bigger profits with the new prices. However, in its most common form, the menu cost is thought of a cost of adjustment that depends on the magnitude of the change. In that way, the cost would also include other eventualities such as concerns about upsetting customers, negotiating new contracts with employees, etc.

A common continuous specification is to assume that a firm's total cost of price adjustment depends in a convex manner on the magnitude of the price change it implements. Specifically, we assume that the *real* cost to a price-setting firm j of changing its nominal price is:

$$\frac{\phi}{2}\left(\frac{p_{jt}}{p_{jt-1}}-1\right)^2$$

where $\phi > 0$ is a convenient scaling parameter, since setting $\phi = 0$ takes us back to flexible-pricing (i.e.: zero menu cost). Note that if the monopolistic-producer decides to set $p_{jt} = p_{jt-1}$ it pays no menu cost since the quadratic term disappears. Instead if it chooses to set p_{jt} different from p_{jt-1} it does incur a cost, which is larger the further away from the original level it is. Last, note that the price-adjustment cost is a real cost, that is, it is denominated in terms of consumption goods.

In the version of the Rotemberg model outlined below we will consumers and two sets of productive firms. A retail firm which operates in a perfectly competitive environment and a continuum of intermediate good producers who have some monopoly power over the good which they manufacture. The details are as follows.

Retail Firms

Assume there exist a representative firm that combines intermediate inputs to produce a good sold in a perfectly competitive market. This firm utilizes the Dixit-Stiglitz aggregator to produce the retail good and maximize profits:

$$\max_{\{y_t, y_{it}\}} \pi_t = p_t y_t - \int_0^1 p_{it} y_{it} \; di$$
$$s.t. \quad : \quad y_t = \left[\int_0^1 y_{it}^{1/\varepsilon} \; di \right]^{\varepsilon}$$

which implies:

$$\max_{\{y_{it}\}} \pi_t = p_t \left[\int_0^1 y_{it}^{1/\varepsilon} \; di \right]^{\varepsilon} - \int_0^1 p_{it} y_{it} \; di$$

Profit maximizing by the retail firm leads to a demand function for any intermediate production good j[6]:

$$y_{jt} = \left(\frac{p_{jt}}{p_t} \right)^{\frac{\varepsilon}{1-\varepsilon}} y_t$$

Intermediate Goods Producers

Assume there exists a continuum $[0,1]$ of intermediate goods producers. We focus on the activities and decisions of one particular intermediate manufacturer j, and make the following assumptions on its production technology: (1) there are no fixed costs; (2) the per-unit production cost of each unit of intermediate output is identical regardless of the scale of production (i.e.: constant returns to scale). Together these assumptions imply that the firm's marginal cost of production is invariant to the quantity it chooses to produce.

Intermediate producers, however, also face a second type of cost, separate from costs associated with physical production: the quadratic menu costs. Given this, we can re-write the nominal profit expression for firm j as:

$$\max_{\{p_{jt}\}} \pi_{jt} = p_{jt} y_{jt} - p_t mc_{jt} y_{jt} - \frac{\phi}{2} \left(\frac{p_{jt}}{p_{jt-1}} - 1 \right)^2 p_t$$

which essentially is total nominal revenues minus total nominal costs. To simplify things we will assume that, from here onwards, all intermediate goods producers will have the same marginal cost, which affords us to drop the subindex j.

Note that in a flexible-price environment it is sufficient to simply maximize (after appropriately substituting in the firm's demand function) the expression above. The menu cost, however, introduces a dynamic element into the firm's optimization problem, an aspect completely absent in the flexible-price benchmark. In fact, this dynamic element to a firm's opti-

[6]See the appendix for details.

mal pricing decision should be thought of as *the* fundamental difference between the sticky and the flexible price set-ups.

Consider the firm's nominal profits in period $t+1$:

$$\pi_{jt+1} = p_{jt+1}y_{jt+1} - p_{t+1}mc_{t+1}y_{jt+1} - \frac{\phi}{2}\left(\frac{p_{jt+1}}{p_{jt}} - 1\right)^2 p_{t+1}$$

It is easy to see that profits in period $t+1$ depend in part on the nominal price p_{jt} charged in period t. This is due to the presence of p_{jt} as part of the period $t+1$ cost of price adjustment. In other words, apart from any physical costs of production, a particular price p_{jt} chosen for period t has consequences, *ceteris paribus*, for both the menu costs the firm will incur in period t as well as the menu costs the firm will have to incur in period $t+1$. Thus in deciding its optimal period-t nominal price p_{jt}, the intermediate good producer must take into account not only its period-t profits but rather its discounted profits across periods t and $t+1$. Specifically, the relevant object to maximize becomes:

$$p_{jt}y_{jt} - p_t mc_{jt}y_{jt} - \frac{\phi}{2}\left(\frac{p_{jt}}{p_{jt-1}} - 1\right)^2 p_t + \frac{\beta}{1+\pi_{t+1}}\left[p_{jt+1}y_{jt+1} - p_{t+1}mc_{t+1}y_{jt+1} - \frac{\phi}{2}\left(\frac{p_{jt+1}}{p_{jt}} - 1\right)^2 p_{t+1}\right]$$

where we have applied a modified form of the discount factor β to period $t+1$ profits. Specifically, the discount factor required here is a nominal discount factor, rather than a real one. Thus in addition to the real discount factor β we need to adjust by the one-period ahead nominal discount factor p_t/p_{t+1}, which is simply $1/(1+\pi_{t+1})$.

Substituting in the demand function for the intermediate good j in both periods, we can re-express the firm's (now dynamic) profit function as:

$$\begin{aligned} & p_{jt}\left(\frac{p_{jt}}{p_t}\right)^{\frac{\varepsilon}{1-\varepsilon}} y_t - p_t mc_{jt}\left(\frac{p_{jt}}{p_t}\right)^{\frac{\varepsilon}{1-\varepsilon}} y_t - \frac{\phi}{2}\left(\frac{p_{jt}}{p_{jt-1}} - 1\right)^2 p_t \\ + & \frac{\beta}{1+\pi_{t+1}}\left[p_{jt+1}\left(\frac{p_{jt+1}}{p_{t+1}}\right)^{\frac{\varepsilon}{1-\varepsilon}} - p_{t+1}mc_{t+1}\left(\frac{p_{jt+1}}{p_{t+1}}\right)^{\frac{\varepsilon}{1-\varepsilon}} - \frac{\phi}{2}\left(\frac{p_{jt+1}}{p_{jt}} - 1\right)^2 p_{t+1}\right] \end{aligned}$$

which after combining some of the p_{jt} terms can be re-written as:

$$\begin{aligned} & p_{jt}^{\frac{1}{1-\varepsilon}} p_t^{\frac{\varepsilon}{\varepsilon-1}} y_t - p_{jt}^{\frac{1}{1-\varepsilon}} p_t^{\frac{2\varepsilon-1}{\varepsilon-1}} mc_t y_t - \frac{\phi}{2}\left(\frac{p_{jt}}{p_{jt-1}} - 1\right)^2 p_t \\ + & \frac{\beta}{1+\pi_{t+1}}\left[p_{jt+1}\left(\frac{p_{jt+1}}{p_{t+1}}\right)^{\frac{\varepsilon}{1-\varepsilon}} y_{t+1} - p_{t+1}mc_{t+1}\left(\frac{p_{jt+1}}{p_{t+1}}\right)^{\frac{\varepsilon}{1-\varepsilon}} y_{t+1} - \frac{\phi}{2}\left(\frac{p_{jt+1}}{p_{jt}} - 1\right)^2 p_{t+1}\right] \end{aligned}$$

The first order condition of this expression with respect to p_{jt} is:

$$\begin{aligned}\left(\frac{1}{1-\varepsilon}\right) p_{jt}^{\frac{\varepsilon}{1-\varepsilon}} p_t^{\frac{\varepsilon}{\varepsilon-1}} y_t \quad &- \quad \left(\frac{\varepsilon}{1-\varepsilon}\right) p_{jt}^{\frac{2\varepsilon-1}{1-\varepsilon}} p_t^{\frac{2\varepsilon-1}{\varepsilon-1}} mc_t y_t - \phi\left(\frac{p_{jt}}{p_{jt-1}} - 1\right)\frac{p_t}{p_{jt-1}} \\ &+ \quad \frac{\beta\phi}{1+\pi_{t+1}}\left(\frac{p_{jt+1}}{p_{jt}} - 1\right)\frac{p_{t+1}}{p_{jt}}\frac{p_{jt+1}}{p_{jt}} = 0\end{aligned}$$

We can see from the expression above that terms arise through the $t+1$ price-adjustment cost because a choice for p_{jt} has consequences for, among other things, the menu costs that will be incurred in later periods. Also, note that if $\phi = 0$, we would have the same first-order condition as in the simple flexible price Dixit-Stiglitz framework.

Equilibrium

Consider a symmetric equilibrium where all intermediate producers utilize the same production technology (and thus have the same marginal cost of production), and face the same elasticity of demand for their product. In such a scenario, every firm would optimally set the same price for their product, blurring the distinction between intermediate and retail goods producers. Imposing symmetry, the first-order condition we just derived becomes:

$$\begin{aligned}\left(\frac{1}{1-\varepsilon}\right) p_t^{\frac{\varepsilon}{1-\varepsilon}} p_t^{\frac{\varepsilon}{\varepsilon-1}} y_t \quad &- \quad \left(\frac{\varepsilon}{1-\varepsilon}\right) p_t^{\frac{2\varepsilon-1}{1-\varepsilon}} p_t^{\frac{2\varepsilon-1}{\varepsilon-1}} mc_t y_t - \phi\left(\frac{p_t}{p_{t-1}} - 1\right)\frac{p_t}{p_{t-1}} \\ &+ \quad \frac{\beta\phi}{1+\pi_{t+1}}\left(\frac{p_{t+1}}{p_t} - 1\right)\frac{p_{t+1}}{p_t}\frac{p_{t+1}}{p_t} = 0\end{aligned}$$

Also, note that $p_t^{\frac{\varepsilon}{1-\varepsilon}} p_t^{\frac{\varepsilon}{\varepsilon-1}} = p_t^{\frac{-\varepsilon}{\varepsilon-1}} p_t^{\frac{\varepsilon}{\varepsilon-1}} = p_t^0 = 1$ and $p_t^{\frac{2\varepsilon-1}{1-\varepsilon}} p_t^{\frac{2\varepsilon-1}{\varepsilon-1}} = p_t^{\frac{1-2\varepsilon}{\varepsilon-1}} p_t^{\frac{2\varepsilon-1}{\varepsilon-1}} = p_t^0 = 1$. Hence the above becomes:

$$\left(\frac{1}{1-\varepsilon}\right)[1-\varepsilon mc_t]y_t - \phi\left(\frac{p_t}{p_{t-1}} - 1\right)\frac{p_t}{p_{t-1}} + \frac{\beta\phi}{1+\pi_{t+1}}\left(\frac{p_{t+1}}{p_t} - 1\right)\frac{p_{t+1}}{p_t}\frac{p_{t+1}}{p_t} = 0$$

Next, using the definition of inflation $1 + \pi_t = p_t/p_{t-1}$ yields:

$$\left(\frac{1}{1-\varepsilon}\right)[1-\varepsilon mc_t]y_t - \phi\pi_t(1+\pi_t) + \frac{\beta\phi}{1+\pi_{t+1}}\pi_{t+1}(1+\pi_{t+1})^2 = 0$$

which simplifies to

$$\left(\frac{1}{1-\varepsilon}\right)[1-\varepsilon mc_t]y_t - \phi\pi_t(1+\pi_t) + \beta\phi\pi_{t+1}(1+\pi_{t+1}) = 0$$

The expression above is the new Keynesian Phillips curve. The idea behind it is that when firms are making their optimal decisions, the period-t inflation rate (a consequence of firms' settings for p_{jt}, which in our symmetric equilibrium is identical to p_t) is linked to the period-t marginal cost of production as well as the rate of inflation that will occur in period $t+1$.

Two features of the new Keynesian Phillips curve (NPC) set it apart from the *classic* Phillips curve (PC). First, the classic PC was a relationship relating contemporaneous events only. The inclusion of the future rate of inflation in the NPC came only after the insights gained by neoclassical theory where expectations about the future play a major role in decision making. Second, the classic PC described a relationship between period-t inflation and period-t unemployment, while NPC articulates the relationship between inflation and the marginal costs of production.

Staggered Price Setting and the Calvo-Yun Model

An alternative way of modeling sticky prices is built on the notion that not all firms are able to change prices at the same time. This could happen due to a plethora of motives like different levels of competition, government regulation or even unionization levels. The basic idea will be that, for whatever reason, different sectors in the economy are able to adjust their prices while others cannot.

In spirit this model will follow the same structure as any modern New Keynesian framework in that there will be a monopolistically-competitive producing sector. Yet, to cover a different case to the one previously described, we will assume that the aggregation will be done directly by the consumer. In other words, households will buy from a wide array of monopolistic good producers and will combine those goods into a consumption basket. This has the natural intuition that individuals enjoy variety and derive utility from consuming a plethora of goods. The set up is as follows:

Households choose consumption, labor and real balances to maximize lifetime utility:

$$\begin{aligned} \max_{\{c_t,n_t,m_t\}_{t=0}^{\infty}} U &= E_0 \sum_{t=0}^{\infty} \beta^t u\left(\mathbb{C}_t, n_t, \frac{m_t}{p_t}\right) \\ s.t. &: \quad w_t n_t + b_{t-1} + T_t \geq \int_0^1 p_t^i c_t^i \ di + q_t b_t \end{aligned}$$

where

1. c_t^i is the real consumption of good i at time t
2. $\mathbb{C}_t$ is an aggregate or good basket such that:

$$\mathbb{C}_t = \left[\int_0^1 (c_t^i)^{\frac{\epsilon}{\epsilon-1}} \ di\right]^{\frac{\epsilon}{\epsilon-1}}$$

3. ϵ is the price elasticity of demand for every good. Note that $\epsilon \rightarrow 0$, implies perfect competition
4. p_t^i is the price of consumption good i at time t
5. T_t are nominal transfers made by government at time t
6. The budget constraint is expressed in nominal terms

Recursive Formulation

If we assume no long-term inflation is zero (i.e.: there is no trend), we can write the above problem as:

To be completed...

Appendix

(I) The Dixit-Stiglitz Framework: A model of Monopolistic Competition

The neoclassical view of the economy is premised on perfect competition in all markets (goods, labor, financial, etc.). Modern New Keynesian models are based on a monopolistically-competitive view where the fundamental idea is that there are many goods that consumers purchase and that they all are, to some degree, imperfect substitutes for each other.

Because firms are able to set their prices, we will often make reference towards how much a firm's chosen price exceeds the cost of production of a given unit of the good. A firm's gross markup is defined as the (per unit) price it charges divided by its marginal cost. Recall that in a perfectly competitive market, market forces would ensure that $p = mc$, implying a gross markup $\mu = 1$. Intuitively, a firm operating under conditions of perfect competition has no scope to earn a (marginal) profit on the goods it sells. In contrast, a firm operating in a monopolistically-competitive context, will earn positive (marginal) profits achieving a gross markup $\mu > 1$.

Imagine a set-up where we have retail firms selling a homogeneous good in a perfectly competitive market. That is we will assume that a given retail firm is completely identical in every respect, including the good it sells, to every other retail firm. Due to this we can assume that there is a representative retail firm.

This retail firm, however, must buy inputs from a plethora of monopolistically-competitive intermediate goods producers. We will assume there is a continuum of intermediate goods, and each good is indexed on the unit interval [0,1]. Furthermore, assume that every good that lies on the unit interval is produced by a unique producer and is imperfectly substitutable with any other of these goods. Since the goods that lie on the unit interval are differentiated products, it is feasible to think about the possibility of some monopoly power.

The competitive sector's production technology is as follows. Retail firms must purchase each of the intermediate inputs and apply some transformation/packaging technology to them and then sell the resulting retail good. This technology is usually referred to as the **Dixit-Stiglitz aggregator**:

$$y_t = \left[\int_0^1 y_{it}^{1/\varepsilon} \right]^{\varepsilon}$$

where y_t is the output in period t of the retailers and y_{it} for $i \in [0, 1]$ is the intermediate good i of which there is an infinite number. The parameter ε measures the curvature of this aggregation (a.k.a: packaging, transformation, etc) technology. Allowing for curvature (i.e.: $\varepsilon > 1$) in the aggregation technology is the basis for the existence of monopolistic competition since curvature implies that firms *must* purchase some of every type of intermediate input[7].

[7]Note that with $\varepsilon = 1$ the resulting linear aggregation technology implies that each of the intermediate goods

Moreover, as it will become clear below, the parameters ε will also denote the gross markup that intermediate goods producers charge.

The profit function of the representative retailer is:

$$p_t y_t - \int_0^1 p_{it} y_{it} di$$

where p_t is the nominal price of the retail good and p_{it} the nominal price of the intermediate input i. The price of any intermediate good is taken as given by the retail firm and so we assume that there are no negotiations between retail and wholesale firms. Inserting the aggregator technology in the above expression yields:

$$p_t \left[\int_0^1 y_{it}^{1/\varepsilon} \right]^{\varepsilon} - \int_0^1 p_{it} y_{it} di$$

The retail's firm only choice variable in the above profit expression are the individual amounts of each intermediate input to buy y_{it} for each $i \in [0,1]$. In other words, given the input and output price it faces, the retail firm makes an optimal choice with respect to each intermediate good in order to maximize its profits.

Taking the first order condition of the profit function with respect to y_{ij} (i.e.: w.r.t. good j) we have:

$$\varepsilon p_t \left[\int_0^1 y_{it}^{1/\varepsilon} di \right]^{\varepsilon - 1} \frac{1}{\varepsilon} y_{jt}^{\frac{1}{\varepsilon} - 1} - p_{jt} = 0$$
$$\Rightarrow p_t \left[\int_0^1 y_{it}^{1/\varepsilon} di \right]^{\varepsilon - 1} y_{jt}^{\frac{1}{\varepsilon} - 1} = p_{jt}$$

We can simplify the above to an expression that can be understood as the demand function for the intermediate input[8]:

$$y_t^{\frac{1}{\varepsilon}} = \int_0^1 y_{it}^{\frac{1}{\varepsilon}} \ di$$

Raising both sides to the power of $\varepsilon - 1$:

$$y_t^{\frac{\varepsilon - 1}{\varepsilon}} = \left[\int_0^1 y_{it}^{\frac{1}{\varepsilon}} \ di \right]^{\varepsilon - 1}$$

Substituting the RHS above into the firm's first order condition yields:

$$p_t y_t^{\frac{\varepsilon - 1}{\varepsilon}} y_{jt}^{\frac{1}{\varepsilon} - 1} = p_{jt}$$

are perfect substitutes for each other.

[8]First cancel the ε terms. Then simplify the terms in square brackets by manipulating the exponents.

We now seek to isolate the term y_{jt}. Combining exponents yields:

$$\begin{aligned} y_{jt}^{\frac{1-\varepsilon}{\varepsilon}} &= \left(\frac{p_{jt}}{p_t}\right) y_t^{\frac{1-\varepsilon}{\varepsilon}} \\ &\Rightarrow \quad y_{jt} = \left(\frac{p_{jt}}{p_t}\right)^{\frac{\varepsilon}{1-\varepsilon}} y_t \end{aligned}$$

which is the demand function for the intermediate input j. The expression satisfies the basic property of having an inverse relationship between the price p_{jt} of the good and its quantity demanded.

As for the monopolistic-competitive producers, we make the following assumptions: (1) there are no fixed costs and (2) the production technology exhibits constant returns to scale. The first assumption assures that the average variable cost of production is equal to the average total cost of production. The second has the implication that the firm's marginal cost is invariant to the quantity that it chooses to produce. Together, both assumptions lead to the convenient consequence that the marginal cost function coincides with the average total cost function. Hence, this implies that the total cost of production can be expressed simply as the quantity produced times the marginal cost of production.

Given these assumptions, the profit function for the intermediate producer can be expressed as:

$$\max_{p_{jt}} \pi_{jt} = p_{jt} y_{jt} - p_t mc_{jt} y_{jt}$$

where p_{jt} is good's j price at time t and p_t is the economy wide price level. The term $mc_{jt} y_{jt}$ denotes the real total cost to wholesaler of j of producing y_{jt} units of output. Last, note that the expression above is written in nominal terms[9].

The monopolistic producer takes as given the demand function it faces when making its profit maximizing choices. In turn the above becomes:

$$\max_{p_{jt}} \pi_{jt} = p_{jt} \left(\frac{p_{jt}}{p_t}\right)^{\frac{\varepsilon}{1-\varepsilon}} y_t - p_t mc_{jt} \left(\frac{p_{jt}}{p_t}\right)^{\frac{\varepsilon}{1-\varepsilon}} y_t$$

clearly the only object under the control of this producer is its good's price p_{jt} [10]. Re-write the above as:

$$\max_{p_{jt}} \pi_{jt} = p_{jt}^{\frac{1}{1-\varepsilon}} p_t^{\frac{\varepsilon}{\varepsilon-1}} y_t - p_{jt}^{\frac{\varepsilon}{1-\varepsilon}} p_t^{\frac{2\varepsilon-1}{\varepsilon-1}} mc_{jt} y_t$$

[9] To convert the real total cost into nominal terms we multiply by p_t (the economy wide price-level), which is simply the price of the "bundled" retail good.

[10] A monopolist can choose price or quantity, but not both.

Taking first order conditions with respect to p_{jt} yields:

$$\left(\frac{\varepsilon}{1-\varepsilon}\right) p_{jt}^{\frac{\varepsilon}{1-\varepsilon}} p_t^{\frac{\varepsilon}{\varepsilon-1}} y_t - \left(\frac{\varepsilon}{1-\varepsilon}\right) p_{jt}^{\frac{2\varepsilon-1}{1-\varepsilon}} p_t^{\frac{2\varepsilon-1}{\varepsilon-1}} mc_{jt} y_t = 0$$

Cancelling some repeated terms yields:

$$p_{jt}^{\frac{\varepsilon}{1-\varepsilon}} p_t^{\frac{\varepsilon}{\varepsilon-1}} - p_{jt}^{\frac{2\varepsilon-1}{1-\varepsilon}} p_t^{\frac{2\varepsilon-1}{\varepsilon-1}} mc_{jt} = 0$$

Multiplying the expression first by $p_{jt}^{\frac{-\varepsilon}{1-\varepsilon}}$ and then by $p_t^{\frac{-\varepsilon}{\varepsilon-1}}$ yields:

$$\begin{aligned} 1 - \varepsilon p_{jt}^{-1} p_t mc_{jt} &= 0 \\ &\Rightarrow p_{jt} = \varepsilon p_t mc_{jt} \\ &\Rightarrow \left(\frac{p_{jt}}{p_t}\right) = \varepsilon mc_{jt} \end{aligned}$$

which implies the optimal real price[11] charged by the intermediate producer is always a simple markup over the marginal cost. Moreover the markup is time-invariant: regardless of the shocks hitting the economy in every period t, the intermediate producer sets its price as a constant markup over the marginal cost.

[11]The nominal too, as it can be seen from the previous expression.